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RE-ENTRANT COSMIC-RAY ALBEDO

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Re-Entrant Cosmic-Ray Albedo

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Abstract. Secondary products of the interaction of cosmic rays with the earth's atmosphere, when projected upward and when of an energy below the Störmer cutoff, are trapped by the earth's magnetic field and guided to an impact with the atmosphere in the hemisphere opposite that of their formation. These particles are called the re-entrant albedo. In the present paper we derive an expression for their intensity and make a simplified computation of a particular case where data are available for comparison.

The transport equation and its solution. As was shown[2] previously [*Ray*, 1960], protons moving in a magnetic field and losing energy in an atmosphere have an intensity which satisfies the equation

$$(d/ds)[(-dE/dx)j] = (-dE/dx)S \qquad (1)$$

where j is the directional intensity in units of (cm^2 sec ster Mev)$^{-1}$, dE/dx is the energy loss given by the Bethe-Bloch formula in Mev/g/cm^2, s is path length along the particle trajectory in centimeters, and S is so defined that $vp^{-2}S\,d\mathbf{x}\,d\mathbf{p}$ is the number of protons added in 1 sec to the volume element in phase space $d\mathbf{x}\,d\mathbf{p}$.

Consider one particular albedo trajectory. Define $s = 0$ at the mirror point in the source hemisphere. Assume that the mirror point is so deep in the atmosphere that we can neglect the downward projected secondaries that are produced above the mirror point, pass through it, and then go upward out of the atmosphere again. Then we may take $j = 0$ at $s = 0$. On integrating (1) we obtain

$$j = (-dE/dx)^{-1}\int_0^s ds(-dE/dx)S \qquad (2)$$

where s is the path length traversed from the mirror point to the point at which the atmospheric density first becomes negligible.

The Bethe-Bloch formula relates uniquely the energy of a proton to its position along the trajectory it follows. Denote by E the energy of the proton as it leaves the atmosphere and by E_0 the energy it must have at the mirror point in order to leave the atmosphere with energy E. Then (2) becomes

$$j = (-dE/dx)^{-1}\int_E^{E_0} dE(S/\rho) \qquad (3)$$

where ρ is the atmospheric density in g/cm^3.

The source function. We want now a way to calculate S/ρ. Let j_p denote the differential directional intensity of star-producing radiation at the source location, n the atmospheric number density, dA an infinitesimal area through which the albedo trajectory passes at right angles, σ the cross section of an atmospheric nucleus for star formation by an incident particle, E' the energy of the incident particle, and $d\Omega'$ the solid angle from within which the incident particle arrives at the target. Then

$$dG = j_p n\, dA\, ds\sigma\, dE'\, d\Omega' \qquad (4)$$

is the number of stars formed per second in $dsdA$ by primaries in $dE'd\Omega'$. Denote by $WdEd\Omega$ the number of secondaries produced in $dEd\Omega$ per star. Then to compute S we must multiply (4) by $(p^2/v)WdEd\Omega$, divide by $dsdAp^2dpd\Omega$, integrate $d\Omega'$ over the upper hemisphere and dE' from the lowest energy primaries present to infinity. The result is

$$S/\rho = (\sigma/m)\int_{2\pi} d\Omega' \int_{E_c}^{\infty} dE' j_p W \qquad (5)$$

where m is the mass of an atmospheric atom and E_c is the least energy for which there exist primary particles at the point and in the direction in question.

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[2] In the reference cited, there are two misprints. In equation 1, $\bar{q}_s$ should be replaced by $\dot{q}_s$, and in equation 3 the coefficient 3 should be replaced by ρ.

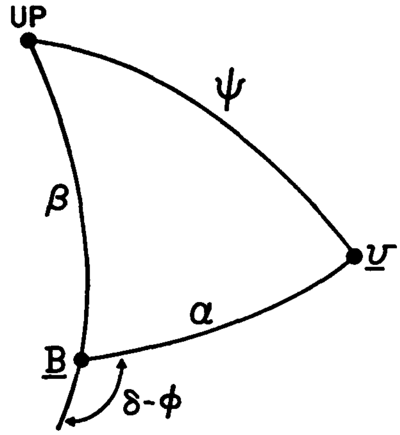

Fig. 1. The coordinate system for describing an albedo trajectory. The magnetic field vector is denoted by **B**.

On putting (5) into (3) we obtain

$$j = (\sigma/m)(-dE/dx)^{-1} \cdot \int_E^{E_0} dE'' \int_{2\pi} d\Omega' \int_{E_c}^{\infty} dE' j_p W \qquad (6)$$

for the differential directional intensity of reentrant albedo with energy E.

The dependence of energy on path length. The integration limit E_0 is obtained from E and the trajectory. We have

$$-(-dE/dx)^{-1}\, dE = \rho\, ds \qquad (7)$$

Now, ρ is a function of h, the height in the atmosphere. Define ψ so that

$$dh = ds \cos \psi \qquad (8)$$

To find an expression for $\cos \psi$, consider Figure 1. From spherical trigonometry,

$$\cos \psi = \cos \alpha \cos \beta - \sin \alpha \sin \beta \cos (\delta - \varphi) \qquad (9)$$

The Larmor frequency is

$$d\, \delta/dt = eB(m_p \gamma c)^{-1} \qquad (10)$$

where m_p is the mass of a proton, γ is $(1 - v^2/c^2)^{-1/2}$, and c is the speed of light. Then from (10),

$$d\, \delta/ds = eB(m_p v \gamma c)^{-1} \qquad (11)$$

From (8), (9), and (11),

$$dh/d\, \delta = m_p v \gamma c (eB)^{-1}[\cos \alpha \cos \beta - \sin \alpha \sin \beta \cos (\delta - \varphi)] \qquad (12)$$

Using $ds = vdt$ together with (7) and (10) we have

$$dE/d\, \delta = -\rho m_p v \gamma c (eB)^{-1} (-dE/dx) \qquad (13)$$

Equations 12 and 13 are a pair of equations to be solved together to obtain E and h as functions of δ. Then α and δ can be used to obtain ψ and the azimuth about the vertical. This is all of the geometric information needed to evaluate the integrals in (6). Besides this, we must select functions to represent W, j_p, and E_c.

An example. We now construct a simplified example. Set

$$W = \lambda(E')^{\nu} \frac{1}{\epsilon} \kappa (2\pi)^{-1} (E/100)^{-\gamma} \qquad (14)$$

All Greek letters appearing here are constants to be used in fitting the Bristol star data, E is the energy of the albedo particle, and E' that of the star-producing particle. From the Bristol data [*Camerini, Davies, Fowler, Franzinetti, Muirhead, Lock, Perkins, and Yekutieli,* 1951] we take the gray track proton multiplicity for a gray track of 100 Mev to be

$$\lambda(E')^{\nu} = 1.15(E')^{0.5}$$

with E' in bev. The range of energy within which gray tracks lie is $\epsilon = 475$ Mev. The fraction of 100-Mev gray tracks projected backward is $\kappa = 0.25$. The factor $(2\pi)^{-1}$ is the reciprocal of the backward solid angle. The dependence on E of κ and λ is given by

$$(E/100)^{-\gamma} = (E/100)^{-1}$$

We also need j_p as a function of angle, energy, and atmospheric depth. No measurements are available in this detail, so we shall construct a suitable function from the spectrum above the atmosphere in the vertical direction [*McDonald and Webber,* 1962] and the dependence on height of the vertical integral intensity [*Clark,* 1952]. We have

$$j_p = j_0 E'^{-\tau}[1 + b\rho H(\cos \omega)^{-1}] \qquad (15)$$

provided that $E' \geq 1$ bev and the square bracket is positive. Otherwise we put $j_p = 0$. We have denoted atmospheric density by ρ, the scale height by H, and the angle between the primary and the vertical by ω. The fitting constants are τ, j_0, and b. The fit we adopt is $b = -0.005/\text{g/cm}^2$ for protons and $b = -0.017/\text{g/cm}^2$ for α particles.

We take $\tau = 1.9$ for protons and $\tau = 1.7$ for α particles.

Assume that α and β in (12) are small enough that we can replace the square bracket with $\cos\alpha \cos\beta$, and that the magnetic field **B** varies so little that $\cos\alpha \cos\beta$ is independent of height. Then (12) and (13) yield

$$-(-dE/dx)^{-1}\, dE = (\cos\alpha \cos\beta)^{-1} \rho\, dh$$

Putting $\rho = \rho_0 \exp(-h/H)$ and $dE/dx = -\mu/E$, this becomes

$$\rho H = (2\mu)^{-1} \cos\alpha \cos\beta (E''^2 - E^2) \qquad (16)$$

We adopt $\mu = 289$ (Mev)^2/g/cm^2. Using (14) and (15) we obtain

$$\int_{E_c}^{\infty} dE' j_p W = \frac{\kappa\lambda}{2\pi\epsilon}[1 + b\rho H/\cos\omega] \cdot (E''/100)^{-\gamma}\left(\frac{\tau - 1}{\tau - \nu - 1}\right) E_c^{\nu} j_T \qquad (17)$$

where

$$j_T \equiv \int_{E_c}^{\infty} dE' j_p$$

is the integral vertical intensity at the top of the atmosphere. For the case we are treating, $j_T = 0.14$ (cm^2 sec ster)$^{-1}$ for protons and 0.02 (cm^2 sec ster)$^{-1}$ for α particles.

To carry out the integration over Ω', assume that j_T and E_c are independent of ω. Then

$$2\pi \int_{-b\rho H}^{1} d\cos\omega [1 + b\rho H/\cos\omega] = 2\pi[1 - b\rho H - b\rho H \ln(-b\rho H)] \qquad (18)$$

Now, E_0 is that value of E'' for which $-b\rho H = 1$. From (18) and (16) this is

$$E_0^2 = E^2 - 2\mu(b \cos\alpha \cos\beta)^{-1} \qquad (19)$$

Putting (17) and (18) into (6) we obtain

$$j = \frac{100\sigma\kappa\lambda}{\mu\epsilon m} \frac{\tau - 1}{\tau - \nu - 1} E_c^{\nu} j_T E \cdot \int_{E}^{E_0} dE''(E'')^{-1} \cdot [1 - b\rho H - b\rho H \ln(-b\rho H)] \qquad (20)$$

Define $\Omega = (-b/2\mu) \cos\alpha \cos\beta E^2$. Expand $\ln(-b\rho H)$ in powers of $(-b\rho H - 1)(-b\rho H + 1)^{-1}$, keeping only the first term. Then (20) becomes

$$j = \frac{100\sigma\kappa\lambda}{2\mu\epsilon m} \frac{\tau - 1}{\tau - \nu - 1} E_c^{\nu} j_T E \cdot \{3 + (1 + 3\Omega^2)(1 - \Omega)^{-1} \cdot \ln[\Omega^{-1}(1 + \Omega)] - 4(1 - \Omega)^{-1} \ln 2\} \qquad (21)$$

For a dip angle of 70°, and assuming that the albedo particle travels along a line of force, $\cos\alpha \cos\beta = 0.94$. For nitrogen, $m = 2.32 \times 10^{-23}$ gram. For protons we take $\sigma_p = 2.25 \times 10^{-25}$ cm^2; for α particles, $\sigma_\alpha = 5 \times 10^{-25}$ cm^2.

Putting the various numerical values in (21) we obtain

$$j = 0.90 \quad (\mathrm{m^2\ sec\ ster\ Mev})^{-1}$$

for proton-produced albedo and

$$j = 0.27 \quad (\mathrm{m^2\ sec\ ster\ Mev})^{-1}$$

for α-particle-produced albedo. The total re-entrant albedo is then

$$j = 1.17 \quad (\mathrm{m^2\ sec\ ster\ Mev})^{-1}$$

These results are for $E = 85$ Mev. This case is appropriate to a measurement by F. B. McDonald and D. A. Bryant at Sioux Falls, South Dakota. They find (private communication) that j lies between 1.0 and 1.1 (m^2 sec ster Mev)$^{-1}$ for particles in the energy range from 70 to 100 Mev.

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